LESSON 1-2 FUNCTIONS, INVERSES, GRAPHING ADJUSTMENTS

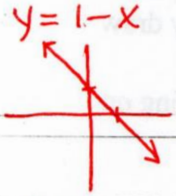
Relation: any set of ordered pairs (any set of points on a graph)

Function: a special type of relation. y is a function of x if for each x-value there is only one y-value. The graph of a function passes the vertical line test. This is written $y = f(x)$.

Domain: the set of all x-values
Range: the set of all y-values
} assuming y is a function of x

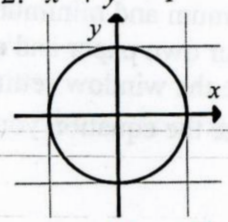
Examples: Determine whether each is a function of x.

1. $x + y = 1$
$y = 1 - x$



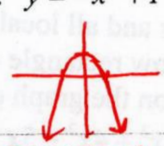
LINE
YES

2. $x^2 + y^2 = 1$



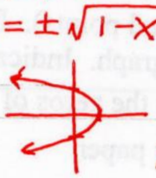
NO $y^2 = 1 - x^2 \rightarrow y = \pm\sqrt{1-x^2}$

3. $y = -x^2 + 1$



PARABOLA
YES

4. $x + y^2 = 1$
$y^2 = 1 - x$
$y = \pm\sqrt{1-x}$



SIDEWAYS PARABOLA
NO

Given: $f(x) = 3x - 1$ and $g(x) = x^2$. Find the following.

5. $f(10) =$
$3(10) - 1$ or 29

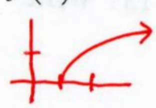
6. $g(x + \Delta x) =$
$(x + \Delta x)^2$
or $x^2 + 2x\Delta x + (\Delta x)^2$
(not suggested)

7. $g(f(x)) =$
$g(3x - 1)$
$= (3x-1)^2$
or $9x^2 - 6x + 1$

8. $(f \circ g)(x) = f(g(x))$
$= f(x^2) = 3x^2 - 1$

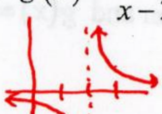
Determine the domain and range for each function.

9. $f(x) = \sqrt{x-1}$



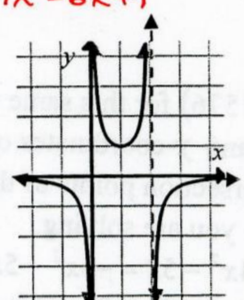
Do: $x \geq 1$
Ra: $y \geq 0$

10. $g(x) = \dfrac{1}{x-2}$



Do: $x \neq 2$
Ra: $y \neq 0$

11.



Do: $x \neq 0, 2$ Ra: $y < 0$, $y \geq 1$

One-to-one Function: a function in which not only is there only one y for each x, but there is also only one x for each y. The graph passes the horizontal line test as well as the vertical line test.

Inverse Function: found by switching x and y and solving for the new y. $f^{-1}(x)$ is the symbol for the inverse of $f(x)$. Only one-to-one functions have inverse functions. Since x and y are switched to produce inverse functions, the domain of f is the range of f^{-1} and vice versa. If (a,b) is in the f function, then (b,a) is in the f^{-1} function.

Examples:

12. Which of the relations in Examples 1-4 is a function with an inverse function? ONLY EX. 1

13. Find the inverse of $f(x)=2x^3-1$. $y=2x^3-1$ $y=\sqrt[3]{\frac{x+1}{2}}=f^{-1}(x)$

① Switch x and y

$x=2y^3-1$

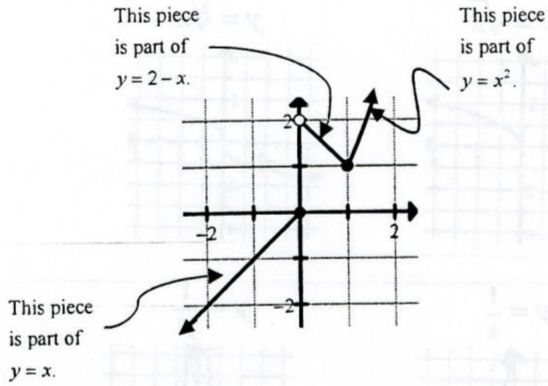
② Isolate the new y

$x+1=2y^3$

$\frac{x+1}{2}=y^3$

Piecewise Function: a function defined differently on different pieces of its domain.

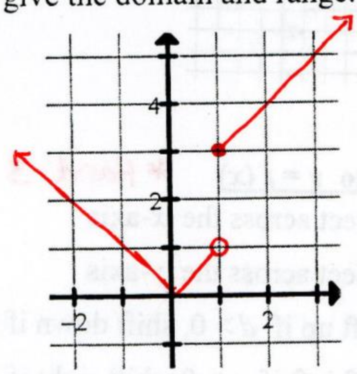
Example: $f(x)=\begin{cases} x, & x\le 0 \\ 2-x, & 0<x<1 \\ x^2, & x\ge 1 \end{cases}$



Examples:

14. Graph this piecewise function and give the domain and range.

$f(x)=\begin{cases} |x|, & x<1 \\ x+2, & x\ge 1 \end{cases}$



Do: all reals

Ra: $y\ge 0$ or $f(x)\ge 0$

Zeros: x-values for which y equals zero.
Conventionally, zeros are written as single values (e.g. $x=2$ or $x=5$) while x-intercepts are written as ordered pairs (e.g. (2,0) or (5,0)).

Find the zeros without using a calculator. use algebraic techniques.

15. $f(x)=x^2-3x-4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

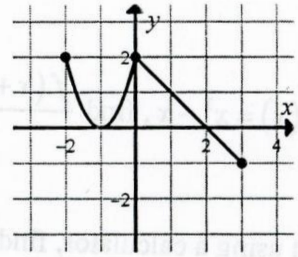
$x=4,-1$

16. $y=\dfrac{x^2-4}{x^2+4}$

$x^2-4=0$ NOT $x^2+4=0$

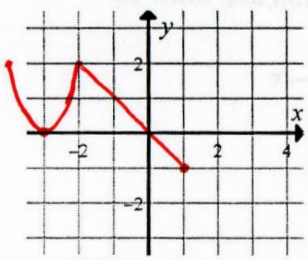
$x^2=4$

$x=\pm 2$



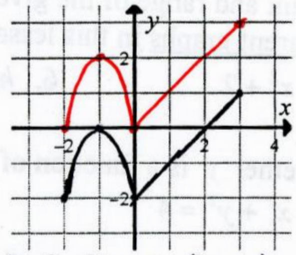
Examples: Use the graph of $y = f(x)$ shown to sketch the following:

17. $y = f(x+2)$



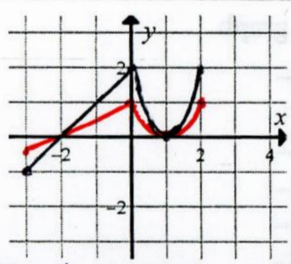
Shift left 2

18. $y = -f(x)+2$



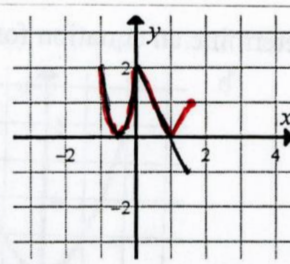
① Reflect (flip) across x-axis : $y = -f(x)$
② Shift up 2 : $y = -f(x) + 2$

19. $y = \frac{1}{2} f(-x)$



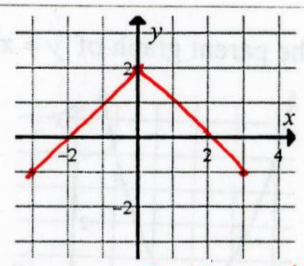
① Reflect across y-axis $y = f(-x)$
② Squeeze vertically by a factor of $\frac{1}{2}$.
(Multiply y-values by $\frac{1}{2}$) $y = \frac{1}{2} f(-x)$

20. $y = |f(2x)|$



① Squeeze horizontally by a factor of $\frac{1}{2}$.
(multiply x-values by $\frac{1}{2}$) $y = f(2x)$
② Reflect the points lying below the x-axis across the x-axis.
$y = |f(2x)|$

21. $y = f(|x|)$



① Eliminate all points to the left of the y-axis.
② Replace the left half of the graph with the mirror image (reflection) of the right half. You should see symmetry to the y-axis.

ASSIGNMENT 1-2

1. If $f(x) = 3x - 2$, find the following.

   a. $f(0)$ b. $f(-3)$ c. $f(b)$ d. $f(x-1)$

2. If $g(x) = \frac{|x|}{x}$, find the following.

   a. $g(2)$ b. $g(-2)$ c. $g(x^2)$